Period \_\_\_\_\_

Date \_\_\_\_\_





#### MATHLINKS: GRADE 6 STUDENT PACKET 13 GEOMETRY

13.1	<ul> <li>Areas of Polygons</li> <li>Derive formulas for the areas of parallelograms, triangles, and trapezoids.</li> <li>Find areas of irregular polygons.</li> <li>Solve real-world and mathematical problems that involve area.</li> </ul>	1
13.2	<ul> <li>Volume</li> <li>Distinguish polyhedra from non-polyhedra.</li> <li>Derive formulas for the volume of a right rectangular prism.</li> <li>Solve real-world and mathematical problems that involve volume.</li> </ul>	12
13.3	<ul> <li>Surface Area</li> <li>Use nets to create polyhedra.</li> <li>Determine the surface areas of polyhedra using nets.</li> <li>Solve real-world and mathematical problems that involve surface area.</li> </ul>	19
13.4	Skill Builders, Vocabulary, and Review	26

### WORD BANK

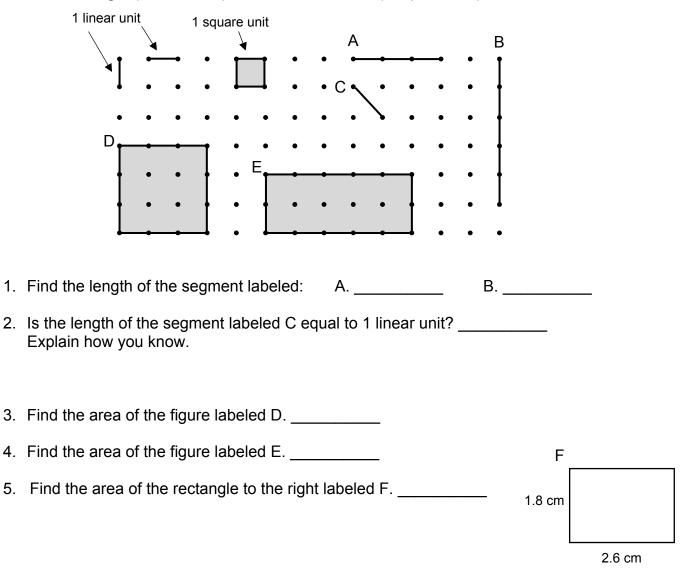
Word or Phrase	Definition or Description	Example or Picture
altitude of a parallelogram		
apex		
area		
base		
net		
polygon		
polyhedron		
prism		
pyramid		
surface area		
volume		

# AREAS OF POLYGONS

Summary	Goals
We will establish formulas for areas of parallelograms, triangles, and trapezoids. We will find areas of irregular polygons. We will apply area formulas to solve problems.	<ul> <li>Derive formulas for the areas of parallelograms, triangles, and trapezoids.</li> <li>Find areas of irregular polygons.</li> <li>Solve real-world and mathematical problems that involve area.</li> </ul>

#### Warmup

One unit of length (1 linear unit) and one unit of area (1 square unit) are defined below.



## **AREA FORMULAS**

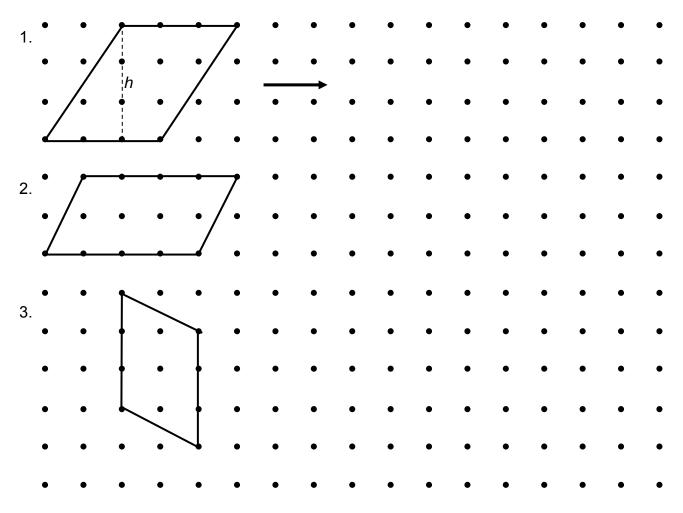
Fill in the table below when instructed by your teacher.

Shape with Definition or Description	Sketch	Area Formula
1. Rectangle:		
2. Square:		
3. Parallelogram:		
4. Rhombus:		
5. Triangle:		
6. Trapezoid:		

## **AREAS OF PARALLELOGRAMS**

A <u>parallelogram</u> is a quadrilateral in which opposite sides are parallel. In a parallelogram, opposite sides have equal length, and opposite angles have equal measure.

- Use a straightedge to draw an altitude of each parallelogram.
- Rearrange the pieces of the parallelogram to form a rectangle with the same area.
- Draw the rectangle next to the corresponding parallelogram.
- Find the area of each figure.



4. Is a rectangle also a parallelogram? Explain.

5. Write in words: The area of a parallelogram is equal to...

6. Write in symbols:  $A_{ }$  = \_\_\_\_\_

## **AREA PRACTICE 1**

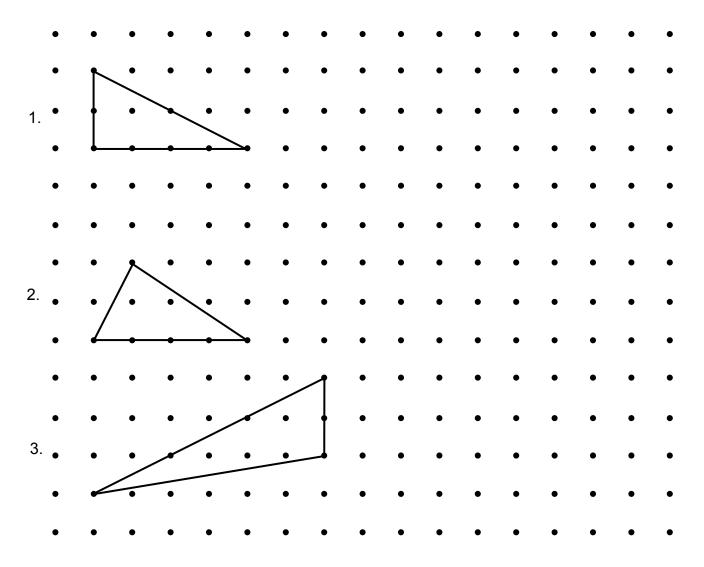
Draw a picture, write an appropriate formula, and substitute to solve each problem.

	Draw a picture, write an appropriate formula, and substitute to solve each problem.				
1.	Find the area of a square tabletop whose side is 25 inches.	2.	Find the area of a parallelogram with a base of 10 inches and a height of 4 inches.		
a.	Sketch the figure:	a.	Sketch the figure:		
b.	Write an appropriate formula:	b.	Write an appropriate formula:		
c.	Substitute:	C.	Substitute:		
d.	What is the area of the square? (Use appropriate units.)	d.	What is the area of the parallelogram? (Use appropriate units.)		
e.	If this square space were a tabletop, would it be big enough for a laptop computer? Explain.	e.	Do you think this parallelogram would be big enough for the base of a free standing basketball hoop? Explain		

## **AREAS OF TRIANGLES**

A <u>triangle</u> is a polygon with three sides.

- Duplicate (or copy) each triangle.
- Rearrange the two triangular pieces to form a rectangle or a parallelogram.
- Draw the rectangle or parallelogram next to the corresponding triangle or on the triangle.
- Find the area of each triangle.



4. Write in words: The area of a triangle is equal to...

5. Write in symbols: A = \_\_\_\_\_

## **AREA PRACTICE 2**

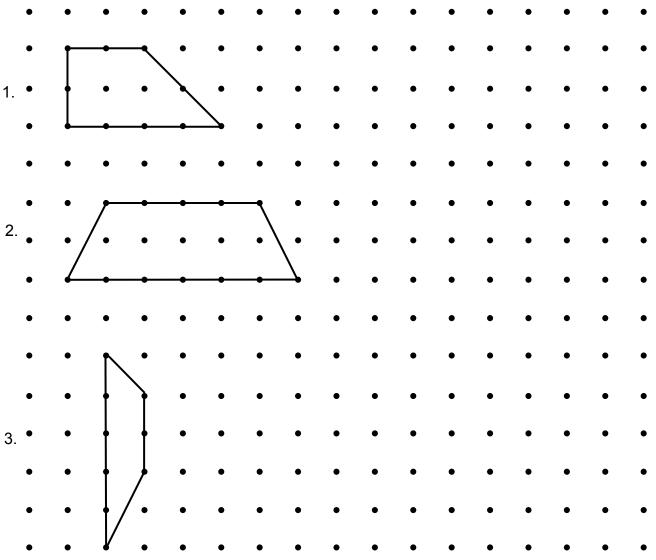
Draw a picture, write an appropriate formula, and substitute to solve each problem.

	iw a picture, write an appropriate formula, ar	10 3	
1.	Find the area of an isosceles triangle with a height of 20 cm and a base of 3 cm.	2.	Madeleine is building a tabletop in the shape of a triangle. The area of the triangle is 1,200 square centimeters, and the base is 60 centimeters.
a.	Sketch a figure:	a.	Sketch two possible figures that meet these conditions:
b.	Write an appropriate formula:	b.	Write an appropriate formula:
C.	Substitute:	C.	Substitute:
d.	What is the area of the triangle? (Use appropriate units.)	d.	What is the height of the triangle? (Use appropriate units.)
e.	Do you think this triangle would make a good outline for a drawing of a clown hat? Explain.	e.	Do you think this tabletop would be a good size and shape for the bottom of a hamster cage? Explain.

## **AREAS OF TRAPEZOIDS**

A trapezoid is a quadrilateral which has at least one pair of parallel sides.

- Duplicate each trapezoid.
- Rearrange the two pieces to form a rectangle or a parallelogram.
- Draw the rectangle or parallelogram next to the corresponding trapezoid or on the trapezoid.
- Find the area of each trapezoid.



4. Write in words: The area of a trapezoid is equal to...

5. Write in symbols:  $A_{\frown}$  = \_\_\_\_\_

## **AREA PRACTICE 3**

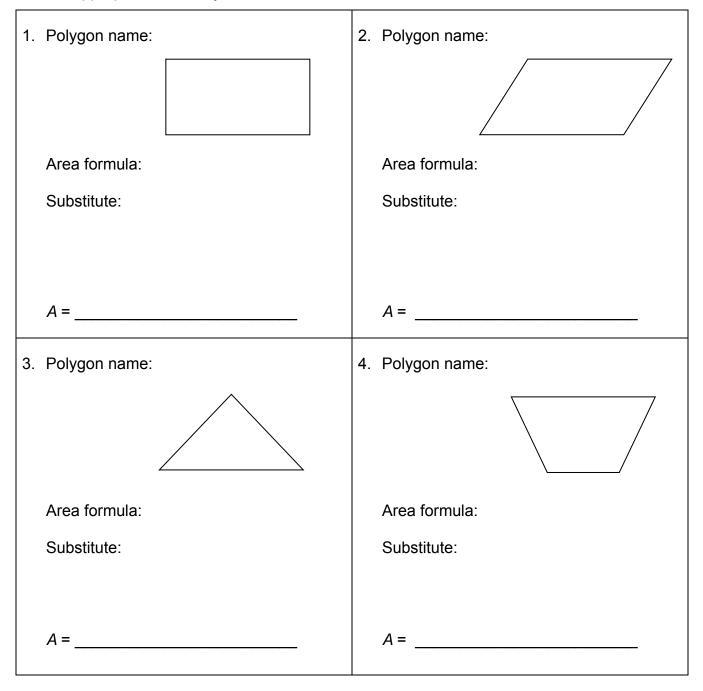
Draw a picture, write an appropriate formula, and substitute to solve each problem.

1.	The height of an isosceles trapezoid measures 2 inches and the bases measure 7 inches and 13 inches. Find its area.	2.	The height of a right trapezoid measures 6 cm and the bases measure 5 cm and 8 cm. Find its area.	
a.	Sketch the figure:	a.	Sketch the figure:	
b.	Write an appropriate formula:	b.	Write an appropriate formula:	
C.	Substitute:	C.	Substitute:	
d.	What is the area of the isosceles trapezoid? (Use appropriate units.)	d.	What is the area of the right trapezoid? (Use appropriate units.)	
e.	Do you think this trapezoid is a good size for a placemat on a table? Explain.	e.	Do you think this trapezoid could be a tabletop for a computer? Explain.	

## **MEASURING TO FIND AREAS OF POLYGONS**

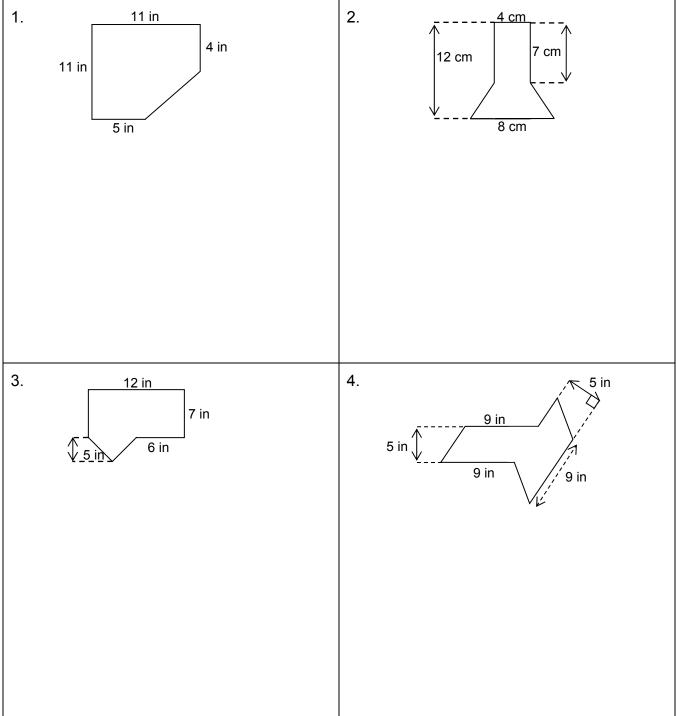
For each problem:

- Identify the polygon and the corresponding area formula.
- Measure and label the relevant dimensions to the nearest tenth of a cm (mm).
- Write the appropriate area formula.
- Substitute values into the formula and evaluate to find the area.
- Use appropriate units in your answers.



## **AREAS OF COMPOSITE FIGURES**

Below are several composite figures, formed from simple figures that do not overlap. Find the area of each composite figure below in square units. Assume any angle that appears to be a right angle is a right angle. Figures are not drawn to scale. Show your work and your answers clearly.



## THE DISPLAY BOARD PROBLEM

This problem is about making a display board for a science fair project. You will use important geometry and measurement skills.

- The display board you create must be a 3 feet by 4 feet rectangle.
- The title sheet on the display board must be cut from an  $8\frac{1}{2}$ -inch by 11-inch piece of paper in the shape of an isosceles trapezoid. One base is  $8\frac{1}{2}$  inches, and the other is 6 inches.
- You put a large picture in the center of the display board inside an isosceles triangle that is also cut from an  $8\frac{1}{2}$ -inch by 11-inch a piece of paper. The base of the triangle is 11 inches and the height is  $8\frac{1}{2}$  inches.
- 1. Make a sample of your title sheet. Label all the measurements you will need to find its area.
- 2. Use graph paper to make a scale drawing of the display board with the title sheet. You may put the title sheet and picture wherever you like.
- 3. Find the area of the display board.
- 4. Find the area of the title sheet.
- 5. Find the area of the triangle with the picture on it.
- 6. Find the area of the remaining space you can use on the display board after the title sheet and triangle with the picture are attached.

Geometry 13.2 Volume					
VOL	UME				
Summary We will establish formulas for the volume of a right rectangular prism in three ways. We will solve real-world and mathematical problems that involve volume.	<ul> <li>Goals</li> <li>Distinguish polyhedra from non-polyhedra.</li> <li>Derive formulas for the volume of a right rectangular prism.</li> <li>Solve real-world and mathematical problems that involve volume.</li> </ul>				
War	mup				
These represent polyhedra.	$\bigwedge$				
These do not represent polyhedra.					
1. Circle the shapes that represent polyhedra.					
2. What do you think are some properties of polyhedra?					
2. What do you think are some properties of polyhedra?					

## **RIGHT RECTANGULAR PRISMS**

A <u>right rectangular prism</u> is a six-sided polyhedron in which all the faces are rectangles. The opposite faces are parallel to each other.

	Prism A	Prism B	Prism C
<ol> <li>Build each right rectangular prism using cubes.</li> </ol>			
<ol> <li>Count the number of cubes needed to build each right rectangular prism.</li> </ol>	cubic units	cu. units	u <sup>3</sup>
<ol> <li>Shade the top layer of cubes for each prism above. Find the volume of the top layer.</li> </ol>			
4. How many layers are in each prism?			

5. How does knowing the volume of the top layer and the number of layers help you determine the number of cubes (volume) in the figure?

6. The area of a rectangle is length $\times$ width $(A = \ell w)$ . Find the area of the top surface for each prism.	square units	sq. units	units²
<ol> <li>Find the height (<i>h</i>) of each prism.</li> </ol>	units	units	units

8. How does knowing the height and the area of the top surface help you determine the number of cubes (volume) in a right rectangular prism?

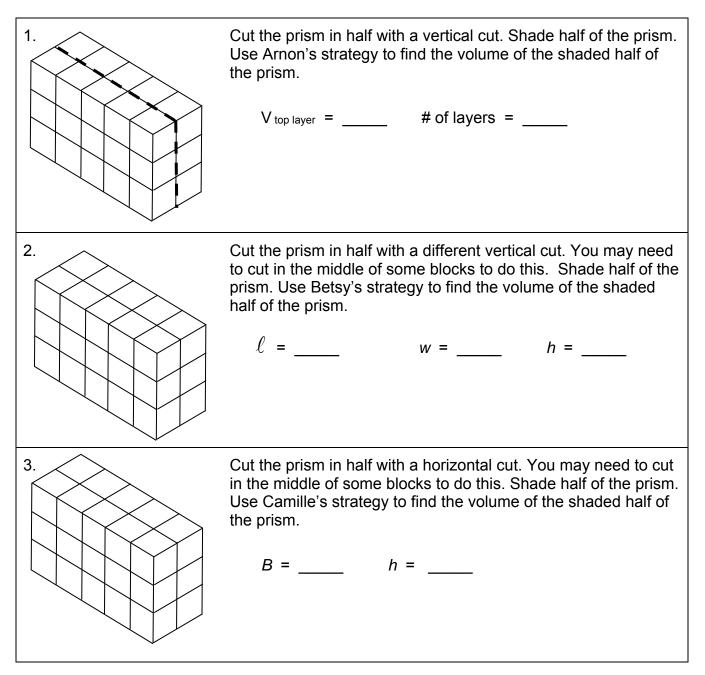
#### STRATEGIES FOR FINDING VOLUME

- 1. Arnon said, "I like to find the volume of a right rectangular prism by finding the volume of the top layer of the prism and multiplying that by the number of layers." Write Arnon's strategy using symbols.
- 2. Betsy said, "To find the volume of a rectangular prism, I like to multiply length, width, and height together." Write Betsy's strategy using symbols.
- 3. Camille said, "I like to call the top surface of the prism the 'base.' To find the volume, I like to multiply the area of the base by the height." Write Camille's strategy using symbols.

	Arnon's strategy	Betsy's strategy	Camille's strategy
4.	Volume top layer	length	Area of base
	Number of layers	width	height
		height	
	V =	V =	V =
5.	V top layer	l	В
	# of layers	w	h
		h	
	V =	V =	V =
6.			

Find the volume of each right rectangular prism using:

## **SLICING A PRISM IN HALF**



- 4. How do the volumes on this page compare to the volume computed on problem 5 on page 14?
- 5. Which volume formula do you prefer to use? \_\_\_\_\_ Why?

## **VOLUME OF PRISMS**

Volume of a right rectangular prism:

 $V = \ell wh \text{ or } V = Bh$ 

Find the volume of each right rectangular prism without counting all of the cubes in the layers individually.

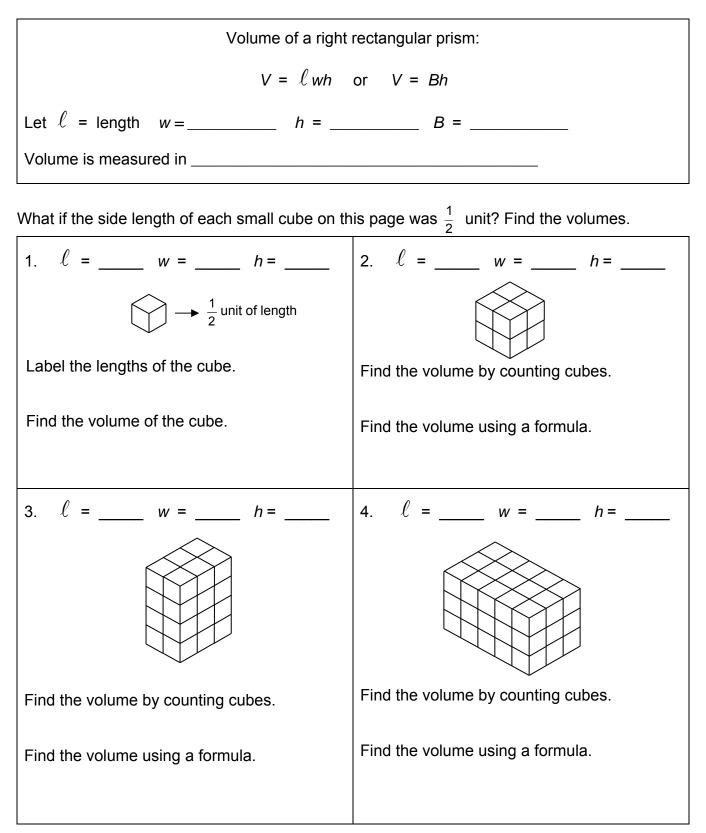
1. Top view of the prism:	2. Top view of the prism:
Height of the prism: 12 units	Height of the prism: $3\frac{1}{2}$ units
Volume of the prism:	Volume of the prism:
3. Top view of the prism:	4. Top view of the prism:
Height of the prism: 7 units	Height of the prism: $\frac{3}{4}$ units
Volume of the prism:	Volume of the prism:
5. Top view of the prism:	6. Top view of the prism:
Height of the prism: 3 units 12 units	Height of the Prism: 2.5 units 9 $\frac{1}{3}$ units 15 units
5.25 units	
Volume of the prism:	Volume of the prism:

# **VOLUME OF PRISMS (Continued)**

For each RIGHT rectangular prism below, use the formula  $V = \ell wh$  or V = Bh. Answer the questions below.

7. What could be the dimensions of this rectangular prism if the volume is 504 cubic units and all lengths are whole numbers?	8. The volume of this rectangular prism is greater than 96 cubic units and less than 168 cubic units. What could be the height of the prism? 8 units 3 units			
$\ell$ = 12 units	h >			
w =	 h <			
h =				
<ul><li>9. The length of the prism is 10 cm.</li><li>The height is half the length. The width is 3 cm less than the length.</li></ul>	<ol> <li>The width of the prism is 3.5 feet. The length is 3 times as long as the width. The height is 1 foot more than the width.</li> </ol>			
Determine the dimensions of the prism:	Determine the dimensions of the prism:			
$\ell$ = w = h =	ℓ = w = h =			
Sketch the prism (hint – use a drawing from above as a model):	Sketch the prism:			
Find the volume:	Find the volume:			

#### WHAT IF?

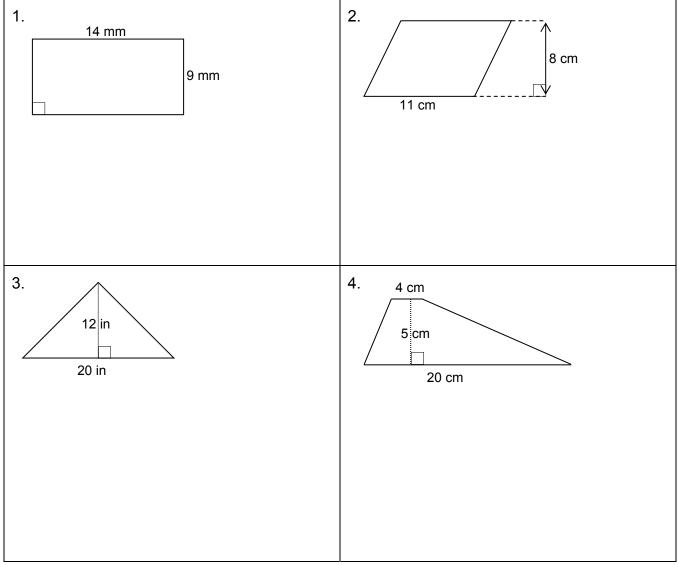


# SURFACE AREA

Summary	Goals
We will construct polyhedra from nets. We will use nets to find surface areas of polyhedra.	<ul> <li>Use nets to create polyhedra.</li> <li>Determine the surface areas of polyhedra using nets.</li> <li>Solve real-world and mathematical problems that involve surface area.</li> </ul>

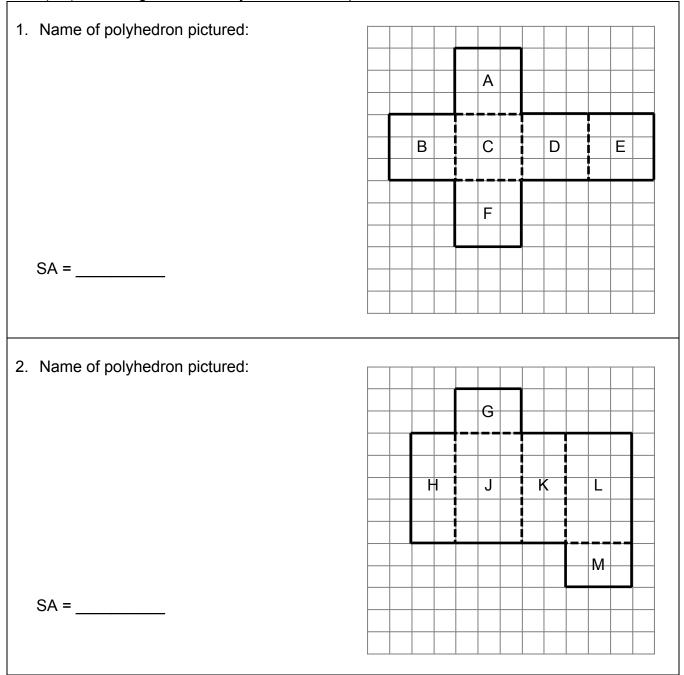
#### Warmup

#### Find the area of each figure.

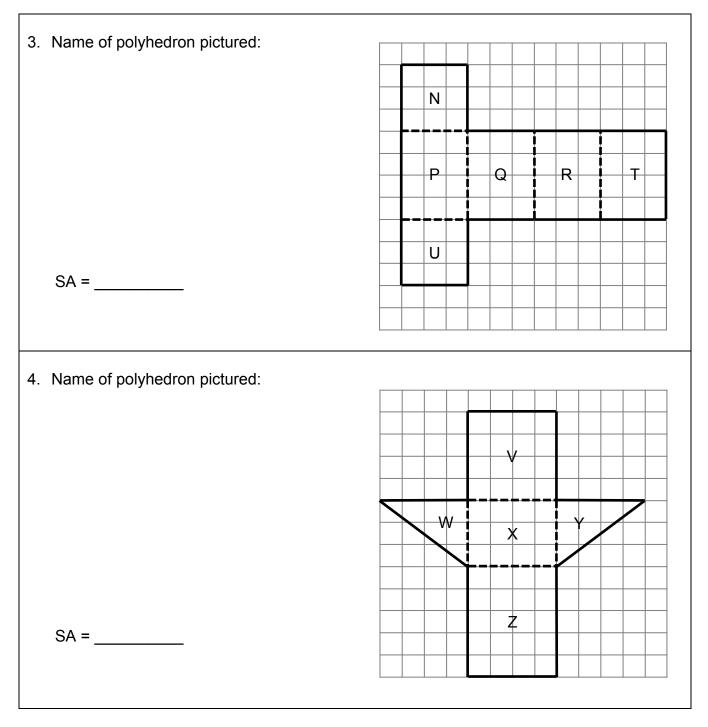


# FINDING SURFACE AREAS OF POLYHEDRA

Your teacher will give you some nets to cut out and tape to create polyhedra. Find the surface area (SA), including units. Show your work or explain.

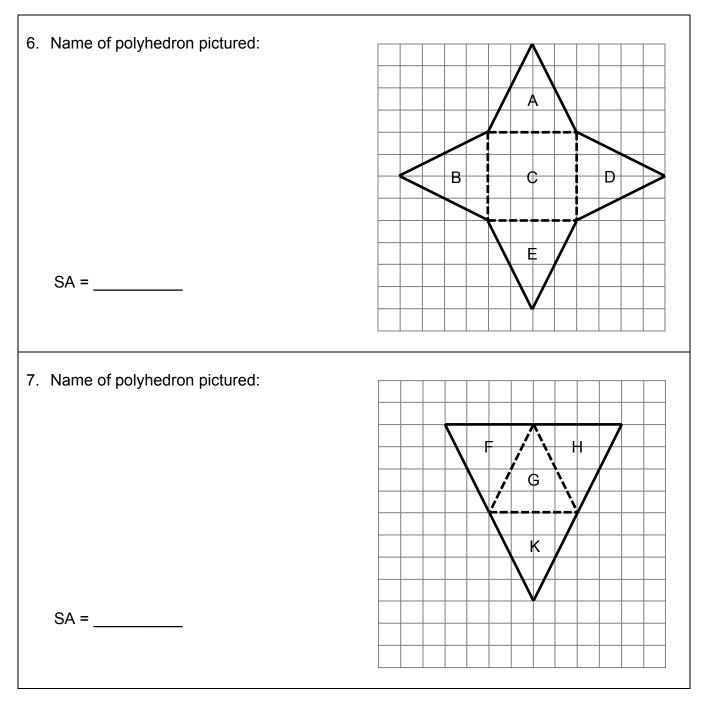


# FINDING SURFACE AREAS OF POLYHEDRA (Continued)



5. Explain how the polyhedron model you built in problem 4 is different from the polyhedra models in problems 1-3.

# FINDING SURFACE AREAS OF POLYHEDRA (Continued)



8. Explain how the polyhedra models on this page are different than those you built in problems 1-4.

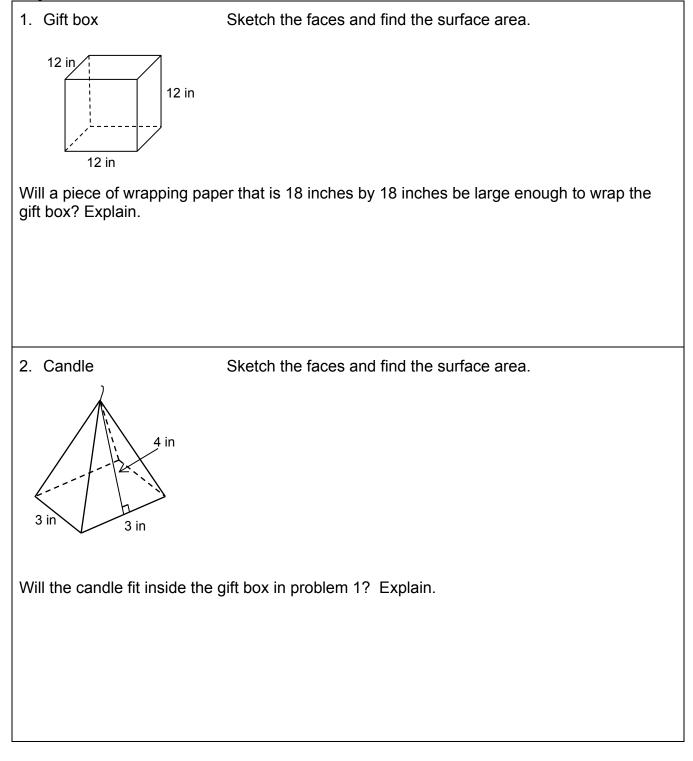
## STRATEGIES FOR SKETCHING POLYHEDRA

Here are a few simple tricks to help you sketch polyhedra. Try these, or share some of your own techniques with your classmates.

1. Start with a "Y." Then add parallel lines to complete the figure. This technique works well when drawing cubes or rectangular prisms. 2. Start by drawing two congruent polygons. Then connect vertices to complete the figure. This technique works well when drawing polyhedra that have two parallel bases (prisms). You can also use dashed lines to indicate edges that cannot be seen if you like. 3. Start by drawing one polygon. Then put a point somewhere on the paper. Connect the point to the vertices of the polygon. Redraw using dashed lines to indicate edges you cannot see. This technique works well when drawing polyhedra with one base and an apex (pyramids).

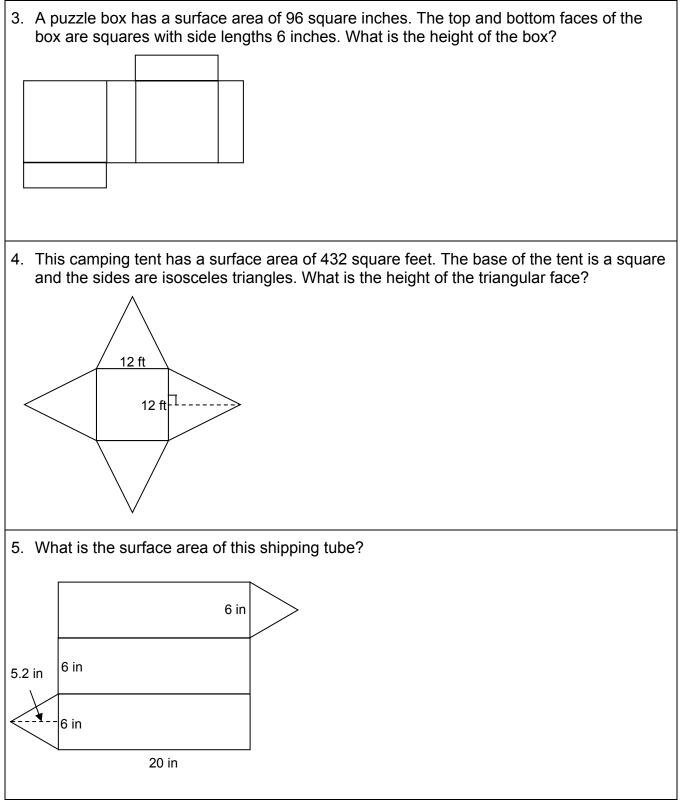
## SURFACE AREA PRACTICE

Sketch the faces for each object. Find its surface area. Then answer the related question. Diagrams are not to scale.



## SURFACE AREA PRACTICE (Continued)

Sketch the polyhedron represented by the net and answer the question.



# SKILL BUILDERS, VOCABULARY, AND REVIEW

#### **SKILL BUILDER 1**

About 1 out of 10 people are left-handed.

1.	What is the ratio of left-handed people to total people?	2.	What is the ratio of left-handed people to right-handed people?
3.	In a group of 70 people, about how many people would you expect to be left-handed?	4.	There are 14 people in a group who are left-handed. Estimate the size of the group.

(For problems 5-10) Kristin walks in a fundraiser for cancer research. She walks at a steady rate, completing 18 miles in 6 hours, and raises \$1,440.

Write a unit rate that represents each of the following statements.

5. miles walked per hour	<ol><li>dollars raised per mile walked</li></ol>	7. dollars raised per hour

Use a table, a double-number line diagram, or equations for problems 8-10. Assume that Kristin walks at a steady, constant rate.

8. How much money will Kristin raise if she walks 12 miles?	9. How long will it take Kristin to walk 12 miles?	10. How far does Kristin have to walk to raise \$2,160?

- 1. Amira cut and used 0.37 meters of a 10 m piece of string. How much string does she have left?
- 2. Mr. Adams drives a school bus 5 days per week, two trips per day. Each trip averages 35.8 miles. How many miles does Mr. Adams drive in one week?

Compute.

3.	5(3 + 2) - 8 • 2	4.	15 ÷ 3 • 4 + 8 – 3	5.	$5 + 8 \cdot 2 - 2^3$

- 6. Miranda's heart beats 3 times every 2 seconds. At that rate, how many times does her heart beat in 1 minute?
- 7. A neighborhood veterinarian treats cats and dogs in her clinic. She keeps track of her patients, and notices that the ratio of cats to dogs is usually about 1:4. Draw a tape diagram to represent this.
  - a. How many cats and how many dogs are in the clinic if there are a total of 45 animals?
  - b. What percent of her patients are cats?

Solve.

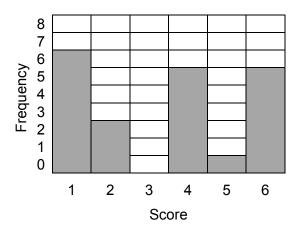
1.	$x + 3\frac{1}{2} = 5\frac{1}{4}$	2.	$w - 1\frac{3}{4} = 3\frac{1}{5}$	3.	$y + 7\frac{2}{3} = 12\frac{5}{8}$
4.	34 <i>y</i> = 2176	5.	4,680 = 24 <i>g</i>	6.	$\frac{w}{154} = 23$

Write <, >, or = to make each statement true.

7.	$3\frac{1}{5} - \frac{17}{5}$	8.	5 <mark>7</mark>	5.87	9.	35 4	8 <sup>3</sup> / <sub>4</sub>	

Each student in a group was challenged to name up to six football teams in 15 seconds. The score of correctly named teams for each student is represented by the graph to the right.

- 10. How many students were involved in the challenge?
- 11. Find the mean, median, mode, and range of the data.



Compute.

1. $\frac{345.9}{0.09}$	2. $\frac{93.84}{0.12}$	

3. Maci wanted to fill an 8 gallon bucket using a two pint plastic container. How many times will Maci need to pour the container to fill the bucket?

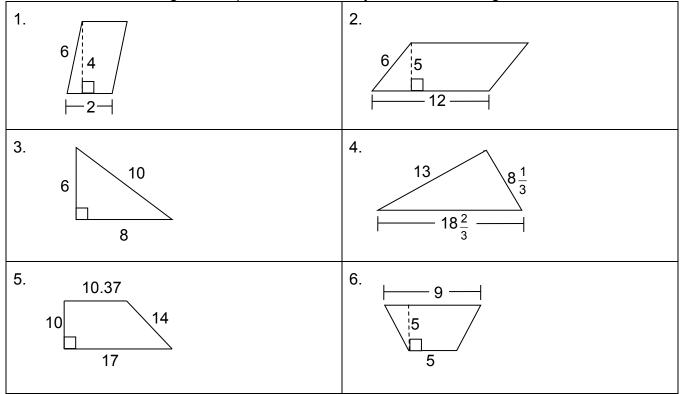
Compute.

4. How many yards equal 40 inches?	5. 3 liters is equal to about how many cups?
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Compute.

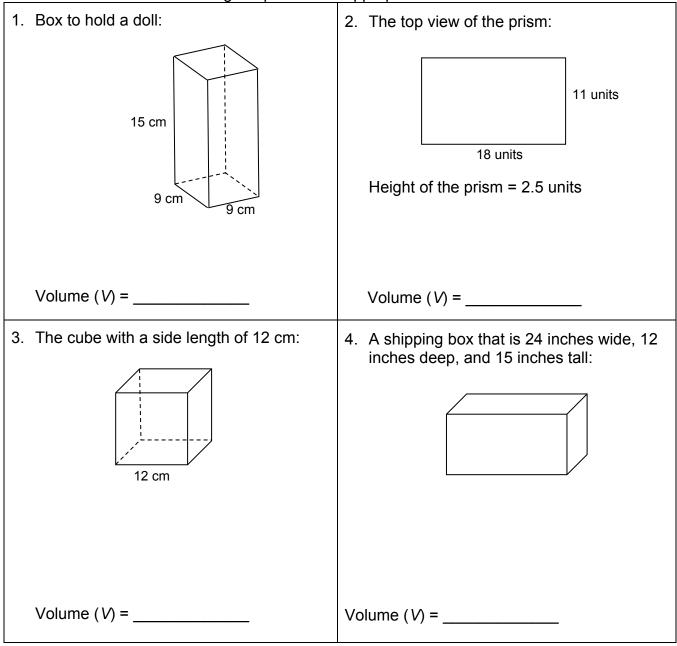
6.	30% of 18 cm	7.	90% of 178 in	8.	12% of 140 days
9.	What is 85% of \$540?	10.	What percent of \$15 is \$30?	11.	What percent is \$12 of \$60?

Find the area of each figure in square units. Show your work. Drawings not to scale.



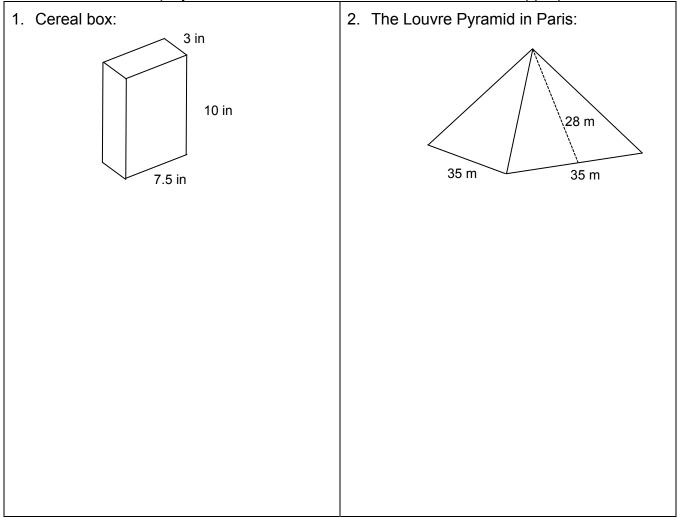
Find the area of each figure in square units. Figures are not drawn to scale.

Find the volume of each rectangular prism. Use appropriate units.

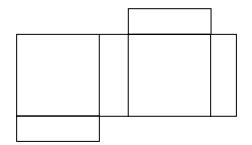


5. The length of the prism is 12 cm. The height is one-third the length. The width is 5 less than the length. Sketch the rectangular prism and find the volume.

Sketch a net for each polyhedron below. Then find its surface area. Use appropriate units.



3. A puzzle box has surface area of 192 square units. The top and bottom faces of the box are squares with side lengths of 8 units. What is the height of the box?

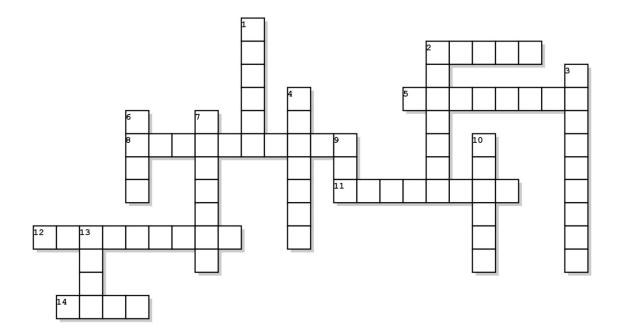


A rectangular prism has a length of 3.5 cm, a width of 4.5 cm, and a height of 5 cm.

1.	Sketch the rectangular prism and label its dimensions.	2.	Sketch the net and label its dimensions.
3.	Find its volume. Use appropriate units. Formula:	4.	Find its surface area. Use appropriate units.

- 5. Sarah has a fish tank shaped like a rectangular prism that is  $18 \text{ cm} \times 20 \text{ cm} \times 16 \text{ cm}$ . What is the volume of the tank?
- 6. If Sarah only fills the tank  $\frac{3}{4}$  of the way, what will be the volume of the water in the tank?

#### FOCUS ON VOCABULARY



#### <u>Across</u>

2 shape of a tissue box

- 5 height = length of \_\_\_\_\_
- 8 closed figure in three-dimensions with polygonal faces
- 11 three-sided polygon
- 12 quadrilateral with at least one pair of parallel sides
- 14 a pre-designated side or face of a figure

#### <u>Down</u>

- 1 rectangle with four equal sides 2 closed figure formed by line segments 3 polygon with four right angles 4 area of faces of a polyhedron: area top of a pyramid 6 7 three-dimensional figure with a polygonal base and triangular sides 9 two-dimensional pattern for a threedimensional figure 10 a measure in cubic units
- 13 a measure in square units

(For word hints, see the word bank and other vocabulary used in this packet.)

## SELECTED RESPONSE

Sho	ow yo	our work on a se	parate s	heet of pa	per and s	selec	t the best a	inswer(s	s).	
1.	Horatio's bedroom floor is 12 feet long and 14 feet wide. Which of the following calculations could he use to calculate the area of his floor? Choose all that apply.									
	A.	12 • 14				В.	14 • 12			
	C.	14 + 12				D.	14 + 14 +	12 + 12		
2.	What is the area of a triangular sticker with a height of 5 cm and a base of 12 cm?									
	A.	17 cm <sup>2</sup>	В.	60 cm <sup>2</sup>		C.	30 cm <sup>2</sup>		D.	25 cm <sup>2</sup>
3.	Find the area of the figure below.									
			3 c	<u>m_</u>			<u>3 cm</u>	1		
				cm			сш			
			cu	4	8 cm		4	сЭ		
			7 0					7 0		
					14 cm			1		
	A.	50 cm <sup>2</sup>	В.	66 cm <sup>2</sup>		C.	98 cm <sup>2</sup>		D.	None of these
4.	Finc	I the surface are	a of the	prism to th	ne right.					
								8 in	6	10 in 7 in
	A.	216 square incl	hes			В.	264 square inches			
	C. 3,360 cubic inches					D.	1,680 cubic inches			
5.	. Choose all of the following that can be used to calculate the volume of the rectangular prism to the right.									

# **KNOWLEDGE CHECK**

Show your work on a separate sheet of paper and write your answers on this page.

#### 13.1 Area of Polygons

For problem 1 and 2, draw a picture, write an appropriate formula, and then solve each problem.

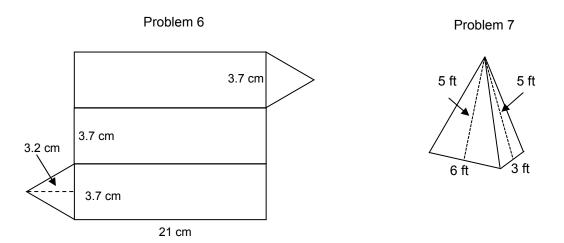
- 1. Find the area of a parallelogram whose base is 76 cm and whose height is 50 cm.
- 2. Find the area of a trapezoid whose bases are 9 meters and 4 meters and whose height is 3 meters.
- 3. Use a ruler to measure the appropriate parts of the composite figure to the right in order to find its area. Measure lengths to the nearest tenth of a cm (nearest mm).

#### 13.2 Volume

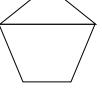
- 4. Find the volume of a shipping box that is 18 inches by 12 inches by 4.5 inches.
- 5. Find the volume of a cube that has a side length of  $3\frac{1}{2}$  inches.

#### 13.3 Surface Area

- 6. The net below folds to make a polyhedron. Find its surface area.
- 7. Find the surface area of the rectangular pyramid below.







## **HOME-SCHOOL CONNECTION**

Here are some problems to review with your young mathematician.

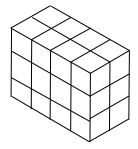
- 1. Sketch two different parallelograms, each with an area of 30 square units, but with different bases and heights. Label the dimensions of each parallelogram.
- 2. Describe the process required to find the area of each of the polygons below.





Use the right rectangular prism to the right for problems 4-6.

- 3. Sketch a net for the prism.
- 4. Find the surface area of the prism.
- 5. Find the volume of the prism.



Parent (or Guardian) Signature

### **COMMON CORE STATE STANDARDS – MATHEMATICS**

#### STANDARDS FOR MATHEMATICAL CONTENT

#### 6.EE.A Apply and extend previous understandings of arithmetic to algebraic expressions.

- 6.EE.2a Write, read, and evaluate expressions in which letters stand for numbers: Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 y.
- 6.EE.2c Write, read, and evaluate expressions in which letters stand for numbers: Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas  $V = s^3$  and  $A = 6 s^2$  to find the volume and surface area of a cube with sides of length s = 1/2.
- 6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.

#### 6.G.A Solve real-world and mathematical problems involving area, surface area, and volume.

- 6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
- 6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = I w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
- 6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

#### STANDARDS FOR MATHEMATICAL PRACTICE

- MP1 Make sense of problems and persevere in solving them.
- MP2 Reason abstractly and quantitatively.
- MP4 Model with mathematics.
- MP5 Use appropriate tools strategically.
- MP6 Attend to precision.
- MP7 Look for and make use of structure.
- MP8 Look for and express regularity in repeated reasoning.



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